# How to Multiply Fractions

Introducing:

- factor
- product
- reciprocal
- inverse
- identity

$$\frac{3}{8} \quad \mathbf{X} \quad 3$$
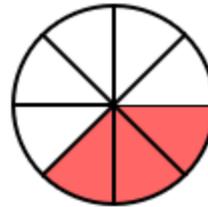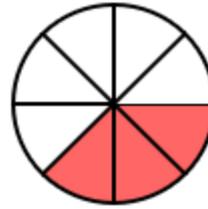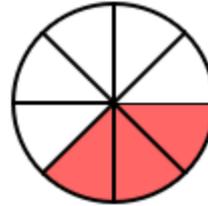
first factor
(number in each row)

second factor
(number of rows)

Multiply Fractions 1

$$\frac{3}{8} \quad \textbf{X} \quad 3$$
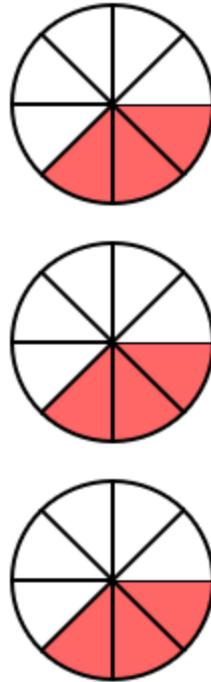
first factor
(number in each row) **second factor**
(number of rows)

The parts of this multiplication example are the first *factor* ³/₈ , and a second *factor* 3. There are 3 rows with ³/₈ in each row.

Multiply Fractions 2
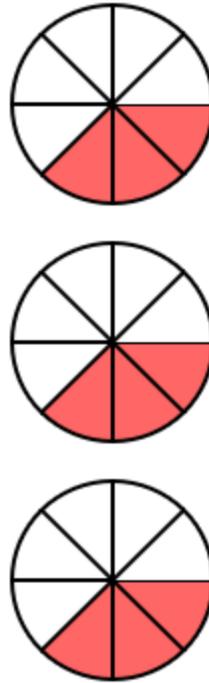
$$\frac{3}{8} \quad \times \quad 3 \quad = \quad 1\frac{1}{8}$$

Multiplication is a form of addition. This picture shows that $\frac{3}{8}$ is added 3 times. The *product* can be found by addition of like amounts:

$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{9}{8}$

Multiply Fractions 3
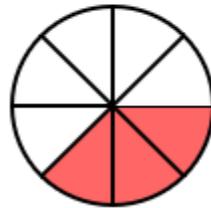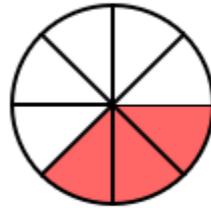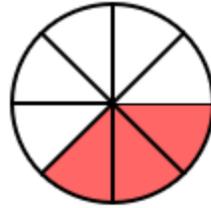
$$\frac{3}{8} \times 3 = \frac{3}{8} \times \frac{3}{1} = \frac{9}{8} = 1\frac{1}{8}$$

first factor
(number in each row) **second factor**
(number of rows)   **Write in fraction form and multiply.**   Simplify.

To calculate the *product,* write both *factors* in fraction form. Then multiply the numerators 3 and 3 for 9 in the *product* numerator and the denominators 8 and 1 for 8 in the *product* denominator.

Multiply Fractions 4
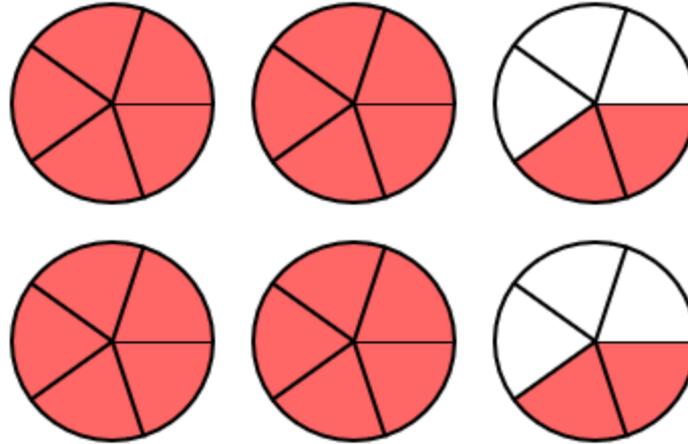
$$2 \; \frac{2}{5} \qquad X \qquad 2 \qquad = \qquad 4 \; \frac{4}{5}$$

It is easy to tell the *product* 4 $\frac{4}{5}$ from this picture. Notice the 4 complete circles and the $\frac{2}{5}$ + $\frac{2}{5}$ circles for a *product* of 4 $\frac{4}{5}$.

Multiply Fractions 5

$$2\,\frac{2}{5} \quad X \quad 2 \quad = \quad \frac{12}{5} \quad X \quad \frac{2}{1} \quad = \quad \frac{24}{5} \quad = \quad 4\,\frac{4}{5}$$

first factor
(number in each row)

second factor
(number of rows)

Write in fraction form and multiply.

Simplify.

To calculate the *product,* write both *factors* in fraction form. Then multiply the numerators 12 and 2 for 24 in the *product* numerator and the denominators 5 and 1 for 5 in the *product* denominator.

Multiply Fractions 6

$$2\frac{2}{5} \quad X \quad 2\frac{0}{1} \quad = \quad \frac{12}{5} \quad X \quad \frac{2}{1} \quad = \quad \frac{24}{5} \quad = \quad 4\frac{4}{5}$$

first factor
(horizontal distance)   **second factor**
(vertical distance)   **Write in fraction form and multiply.**   **Simplify.**

The same example, 2 ²/₅ x 2 using a rectangular array. The first *factor* 2 ²/₅, is shown by the red arrow - the horizontal distance. The second *factor* 2, is shown by the blue arrow - the vertical distance from the bottom. The *product*, 4 ⁴/₅ is enclosed by the yellow rectangle.

Multiply Fractions 7
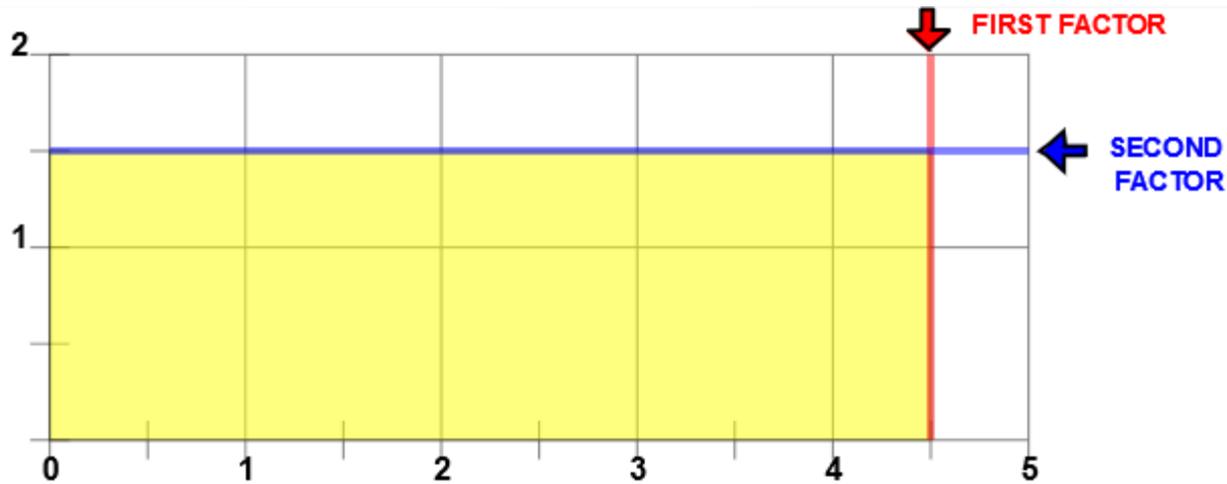
$$4 \frac{1}{2} \quad \times \quad 1 \frac{1}{2} \quad = \quad \frac{9}{2} \quad \times \quad \frac{3}{2} \quad = \quad \frac{27}{4} \quad = \quad 6 \frac{3}{4}$$

first factor — **second factor** — **Write in fraction form and multiply.** — **Simplify.**
(horizontal distance) **(vertical distance)**

This rectangular array shows the product of 4 ¹/₂ and 1 ¹/₂. Notice how each factor has been written in fraction form before multiplying. You can see in the picture that there are 27 fourths.

Multiply Fractions 8

Both *factors* are greater than 1. The *product* is greater than 4 x 1 but less than 5 x 2 by rounding down and rounding up both *factors*. So the product 6 3/4 makes sense.

Multiply Fractions 9
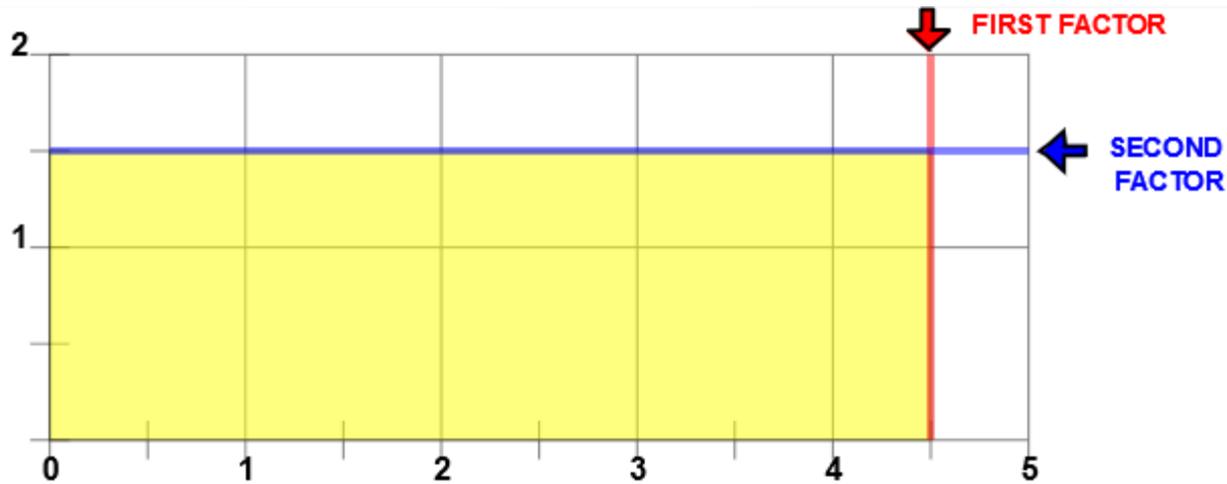
You can tell by the picture from the previous example that there are 4 whole units, five $\frac{1}{2}$ units, and one $\frac{1}{4}$ units. The sum of the units is $4 + \frac{5}{2} + \frac{1}{4} = 6\frac{3}{4}$.

Multiply Fractions 10
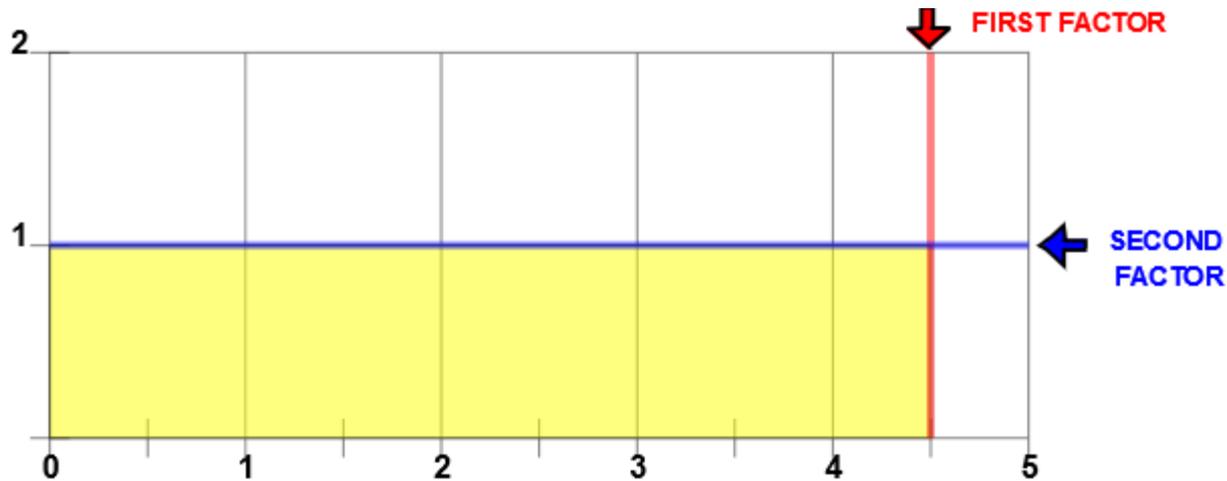
$$4 \frac{1}{2} \quad \times \quad 1 \frac{0}{1} \quad = \quad 4 \frac{1}{2}$$

first factor
(horizontal distance)   **second factor**
(vertical distance)   Multiplying by 1

The second *factor* has been decreased to 1. The *product* has been decreased to 4 $^1/_2$.

Multiply Fractions 11
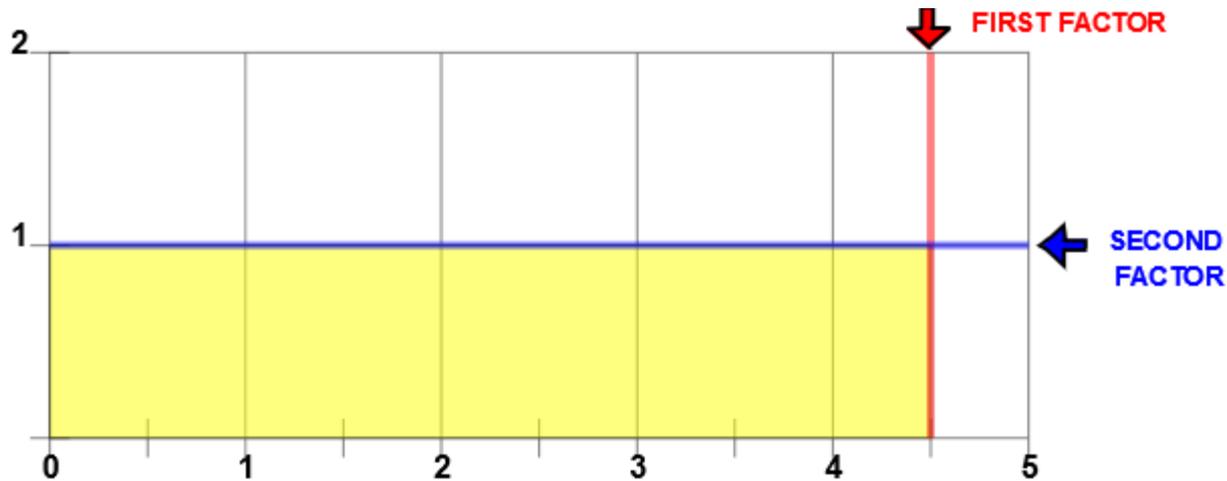
$$4 \frac{1}{2} \quad \times \quad 1 \frac{0}{1} \quad = \quad 4 \frac{1}{2}$$

first factor
(horizontal distance)

second factor
(vertical distance)

Multiplying by 1

When 1 is used as a *factor*, the *product* is equal to the other *factor*. One is called the *identity* for multiplication.

Multiply Fractions 12
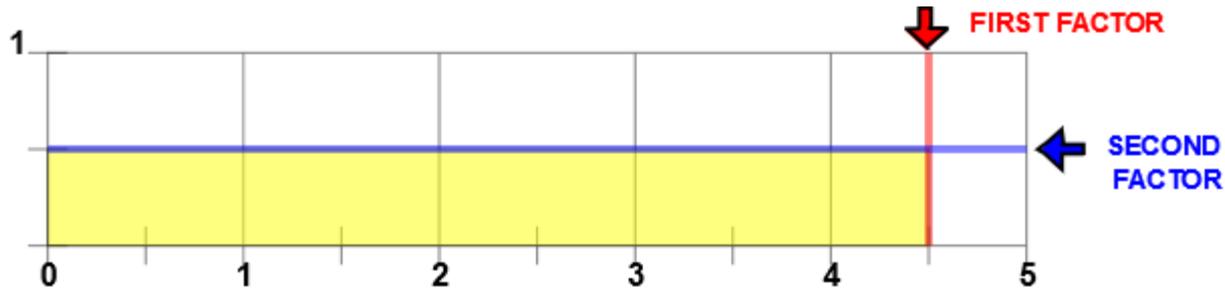
$$4\ \frac{1}{2}\quad \text{X} \quad 0\ \frac{1}{2}\quad = \quad \frac{9}{2}\quad \text{X}\quad \frac{1}{2}\quad = \quad \frac{9}{4}\quad = \quad 2\ \frac{1}{4}$$

first factor
(horizontal distance)   **second factor**
(vertical distance)   **Write in fraction form and multiply.**   Simplify.

The second *factor* has been decreased to ¹/₂. Notice the *product* has been decreased to 2 ¹/₄. When one of the *factors* is smaller than 1, the *product* is smaller than the other *factor*.

Multiply Fractions 13
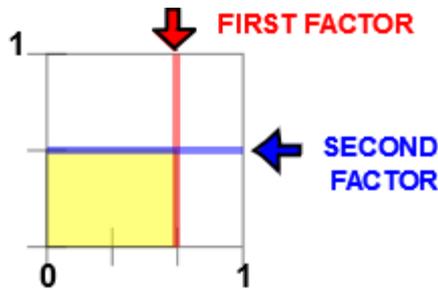
$$0 \; \frac{2}{3} \quad X \quad 0 \; \frac{1}{2} \quad = \quad \frac{\cancel{2}^{1}}{3} \quad X \quad \frac{1}{\cancel{2}_{1}} \quad = \quad \frac{1}{3} \quad = \quad \frac{1}{3}$$

first factor
(horizontal distance) **second factor**
(vertical distance) Write in fraction form, cancel, and multiply. Simplify.

Both *factors* are less than 1. The *product* $\frac{1}{3}$ is smaller than either *factor*.
Notice that the 2 in $\frac{1}{3}$ and the 2 in $\frac{1}{3}$ are canceled. See the canceling
demonstration in this web site at visualfractions.com/cancel/

Multiply Fractions 14
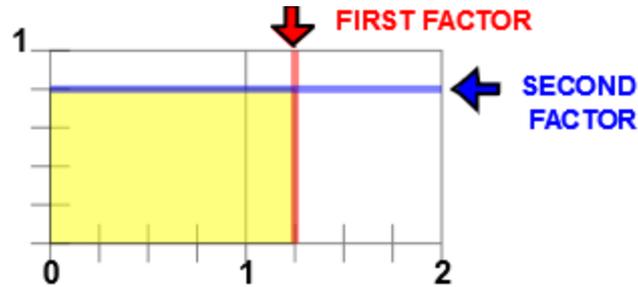
$$1\frac{1}{4} \quad \times \quad 0\frac{4}{5} \quad = \quad \frac{\cancel{5}^{1}}{\cancel{4}_{1}} \quad \times \quad \frac{\cancel{4}^{1}}{\cancel{5}_{1}} \quad = \quad \frac{1}{1} \quad = \quad 1$$

first factor (horizontal distance)   **second factor** (vertical distance)   **Write in fraction form, cancel, and multiply.** Simplify.

The *factors* 1 $^1/_4$ and $^4/_5$ are *reciprocals*. As you can see, multiplying $^5/_4$ by $^4/_5$ gives a *product* of 1. If you are asked to *invert* or write the *reciprocal* of $^5/_4$ you will write $^4/_5$.

Multiply Fractions 15
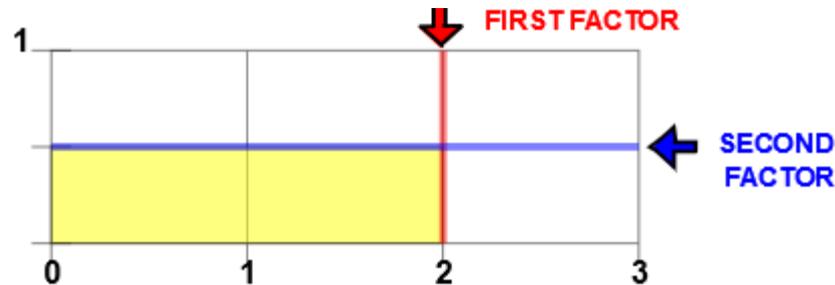
FIRST FACTOR

SECOND FACTOR

$$2\ \frac{0}{1}\quad \times \quad 0\ \frac{1}{2}\quad =\quad \frac{\overset{1}{\cancel{2}}}{1}\quad \times \quad \frac{1}{\underset{1}{\cancel{2}}}\quad =\quad \frac{1}{1}\quad =\quad 1$$

first factor
(horizontal distance)   **second factor**
(vertical distance)   Write in fraction form, cancel, and multiply. Simplify.

To find the *reciprocal* of a fraction, replace the denominator with the numerator and the numerator with the denominator. The *reciprocal* or *inverse* of $^2/_1$ is $^1/_2$.

Multiply Fractions 16
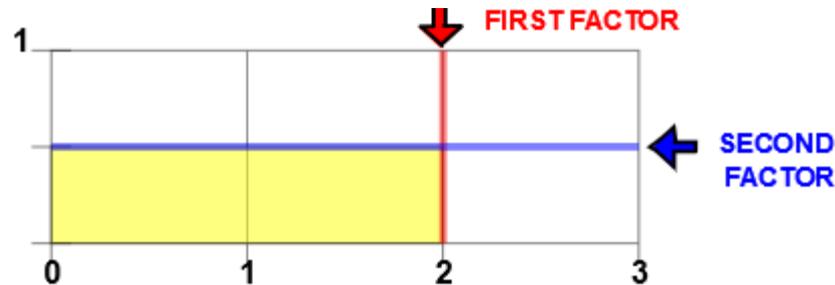
$$2\,\frac{0}{1} \quad \times \quad 0\,\frac{1}{2} \quad = \quad \frac{\cancel{2}^{1}}{1} \quad \times \quad \frac{1}{\cancel{2}_{1}} \quad = \quad \frac{1}{1} \quad = \quad 1$$

**first factor
(horizontal distance)** **second factor
(vertical distance)** **Write in fraction form, cancel, and multiply. Simplify.**

 The picture shows 3 square units. Two $\frac{1}{2}$ squares are selected. This gives a first *factor* of 2 and a second *factor* of $\frac{1}{2}$. Added together, $\frac{1}{2}$ and $\frac{1}{2}$ squares give a *sum* of 1 unit.

 The *factors* 2 and $\frac{1}{2}$ are reciprocals because their *product* is 1.

Multiply Fractions 17

$$2 \frac{2}{3} \quad \times \quad 1 \frac{1}{2} \quad = \quad ?$$

first factor
(horizontal distance)

second factor
(vertical distance)

What is the product of 2 ²/₃ and 1 ¹/₂ ?

Multiply Fractions 18

$$2 \frac{2}{3} \quad \times \quad 1 \frac{1}{2} \quad = \quad \frac{\cancel{8}^{4}}{\cancel{3}_{1}} \quad \times \quad \frac{\cancel{3}^{1}}{\cancel{2}_{1}} \quad = \quad \frac{4}{1} \quad = \quad 4$$

first factor
(horizontal distance)   **second factor**
                (vertical distance)   **Write in fraction form, cancel, and multiply.** Simplify